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An expression for the RBE of neutrons as a function of neutron energy

Thomas E Blue†, Jeffrey E Woollard†, Nilendu Gupta‡ and John F Greskovich Jr‡

† Nuclear Engineering Program, The Ohio State University, Columbus, OH 43210, USA
‡ Biomedical Engineering Department, The Ohio State University, Columbus, OH 43210, USA

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Abstract. The goal of this paper is to develop a relationship between a neutron RBE and neutron energy, E_n, which can be used to design neutron sources for BNCT. In an earlier calculation of a neutron RBE as a function of E_n, we approximated the contribution to a total neutron RBE, $\mathrm{RBE}_t(E_n)$, arising from $^{14}N(n, p)^{14}C$ reactions. In this paper, we recalculate $\mathrm{RBE}_t(E_n)$, accounting more exactly for the contribution to $\mathrm{RBE}_t(E_n)$ from $^{14}N(n, p)^{14}C$ reactions.

1. Introduction

Beam design for boron neutron capture therapy (BNCT) is evolving from being based on the physical characteristics of the beams in air to being based on the biological effects of the beams in human heads (Yanch and Harling 1993). It has been recognized for some time that the physical characteristics of the beams in air lack clinical relevance. However, more clinically relevant parameters based on biological effects in patients' heads, besides being complicated (through their dependence on absorbed dose distributions) by their dependence on head dimensions, necessarily incorporate factors relating absorbed doses in tissues to biological effects. These factors, such as relative biological effectivenesses (RBEs) and compound factors (Gahbauer *et al* 1992) were, in the judgment of most beam designers, not known in the past with sufficient accuracy to justify the added complexity of basing beam designs on parameters based on dose distributions in head phantoms.

However, in preparation for human clinical trials of BNCT, normal tissue tolerance studies in dogs have been carried out recently in the epithermal neutron beams at the Brookhaven Medical Research Reactor (BMRR) and at Petten (Gavin 1993, Siefert *et al* 1993). These studies have provided spectrum-averaged neutron RBEs for clinically relevant endpoints. In order to understand the differences between the measured spectrum-averaged neutron RBEs at the BMRR and Petten, we are investigating an RBE for neutrons as a function of neutron energy (E_n). Our ultimate goal is to develop a relationship between a neutron RBE and E_n that is normalized to the measured spectrum-averaged neutron RBEs at the BMRR and Petten, and which can be used to guide our design of an accelerator neutron source for BNCT. Hopefully, the RBE we develop will be useful to other researchers involved in BNCT beam design, and in the interpretation of BNCT experiments. In particular, we hope to elucidate the relationship between an RBE for the nitrogen component of the dose, RBE^N, and an RBE for the proton recoil component of the dose, RBE^H, since in BNCT the absorbed dose for these components is often calculated separately (Nigg 1994), and often inconsistent RBEs are individually applied to these dose components.

In an earlier calculation (Blue *et al* 1993) of an RBE for neutrons as a function of E_n, the proton fluence resulting from the $^1H(n, n')^1H$ reaction in an infinite tissue medium was used to calculate $RBE^H(E_n)$, the hydrogen component of a total RBE. Then $RBE^H(E_n)$ was combined with an approximation for the component of the RBE resulting from the $^{14}N(n, p)^{14}C$ reaction, $RBE^N(E_n)$, to obtain a relation between a total RBE, $RBE_t(E_n)$, and E_n. In approximating $RBE^N(E_n)$ we assumed that, since neutrons of energy E_n scatter with hydrogen nuclei to create recoil protons with an average kinetic energy ($\langle E_p \rangle$) equal to $E_n/2$, $RBE^N(E_n)$ is simply equal to $RBE^H(E_n)$ evaluated at $E_n = 2E_p^N$, where E_p^N is the kinetic energy of the proton from the $^{14}N(n, p)^{14}C$ reaction.

In this paper, a more exact calculation of an RBE of neutrons as a function of neutron energy is presented. First, an expression for the energy distribution of the proton fluence from the $^{14}N(n, p)^{14}C$ reaction in an infinite tissue medium is written. This expression is then used to calculate an expression for $RBE^N(E_n)$. Finally, the expression for $RBE^N(E_n)$ is combined with the expression for $RBE^H(E_n)$ from our previous paper (Blue *et al* 1993) to yield $RBE_t(E_n)$. In calculating $RBE_t(E_n)$, it must be remembered than an RBE is specific for a given experiment. The RBE calculated in this paper is normalized to a clinically relevant endpoint in dogs. However, it is based, somewhat inconsistently, on a measured relationship between an RBE and the linear energy transfer of charged particles for an endpoint of 10% survival for cultured cells of human origin, which were irradiated *in vitro*.

2. Background

Previously we have determined an expression for the energy distribution of the fluence $\Phi(E_p)$ (protons per unit area per unit energy) at a point, for protons born uniformly throughout an infinite medium, at energy E_0, with source strength $s(E_0)$ protons per unit volume per unit energy (Blue *et al* 1993):

$$\Phi(E_p) = \int_0^\infty dr \int_{E_p}^{E_n} \frac{s(E_0)\delta\{E_p - \Re^{-1}[\Re(E_0) - r]\}}{|(d/dE_0)\{\Re^{-1}[\Re(E_0) - r]\}|} \, dE_0. \tag{1}$$

For protons born uniformly throughout an infinite medium, as a consequence of isotropic (in the centre of mass system) scattering on hydrogen $\Phi(E_p) = \Phi^H(E_p)$, where

$$\Phi^H(E_p) = [\Phi(E_n)\Sigma_s^H(E_n)/\langle -dE/dx \rangle|_{E_p}](1 - E_p/E_n) \qquad E_p < E_n \tag{2}$$

$\Phi(E_n)$ is the neutron fluence (neutrons per unit area) at neutron energy E_n, $\Sigma_s^H(E_n)$ is the macroscopic scattering cross-section for hydrogen in tissue, and $\langle -dE/dx \rangle$ is the average proton energy loss per unit pathlength travelled.

3. Analysis

3.1. $\Phi^N(E_p)$

For neutrons interacting with nitrogen via the $^{14}N(n, p)^{14}C$ reaction, the source strength of protons in (1) becomes

$$s(E_0) = \Phi(E_n)\Sigma_a^N(E_n)\delta(E_0 - E_p^N) \tag{3}$$

where $\Sigma_a^N(E_n)$ is the macroscopic cross-section for the $^{14}N(n, p)^{14}C$ reaction and E_p^N is the kinetic energy of the proton. Since the cross-section for the $^{14}N(n, p)^{14}C$ reaction decreases as $1/\sqrt{E_n}$, most of the protons result from thermal neutron interactions with nitrogen. The Q value for this reaction is 630 keV. The resulting proton has an energy of 590 keV, while the recoil ^{14}C nucleus has an energy of 40 keV (Kobayashi and Kanda 1982). In this paper, the value of E_p^N is set equal to 590 keV, its value for the reaction induced by thermal neutrons. Following a procedure similar to that used by Blue *et al* (1993) for deriving $\Phi^H(E_n)$, we obtain

$$\Phi^N(E_p) = \Phi(E_n)\Sigma_a^N(E_n)/(-dE/dx)|_{E_p} \qquad 0 < E_p < E_p^N. \tag{4}$$

To evaluate $\Phi^N(E_p)$, we used the same values for $\langle -dE/dx \rangle$ as were used by Blue *et al* (1993).

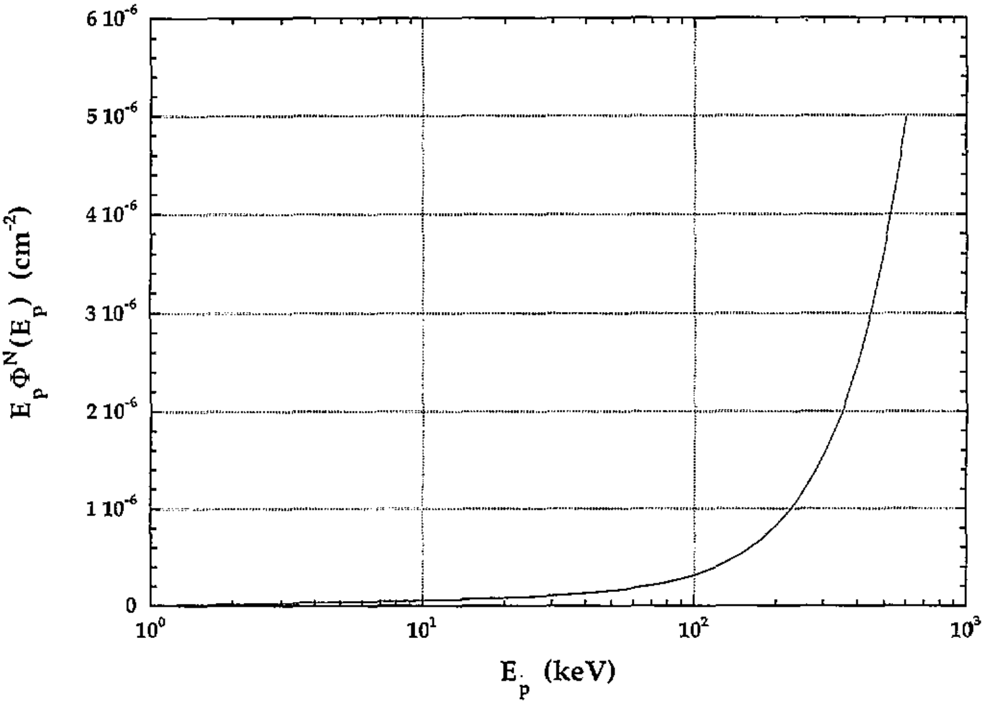

Figure 1. $E_p\Phi^N(E_p)$ (in cm^{-2}) versus E_p (in keV) for protons in tissue for thermal neutrons ($E_n = 0.0253$ eV) assuming $\Phi(E_n) = 1$ cm^{-2}.

Figure 1 is a graph of $E_p\Phi^N(E_p)$ against E_p in tissue for thermal neutrons (E_n = 0.0253 eV), with $\Phi^N(E_p)$ calculated using (4), assuming $\Phi(E_n) = 1$ cm^{-2}. Also, in calculating $\Sigma_a^N(E_n)$ in (4), it was assumed that the tissue has a mass density (ρ) of 1.07 g cm^{-3} (AAPM 1980) and that the tissue is 3.5% nitrogen by weight (ICRU 1976). The shape of $\Phi^N(E_p)/\Phi(E_n)$ is independent of E_n. Only the magnitude of $\Phi^N(E_p)/\Phi(E_n)$ is affected by changes in E_n, as a consequence of the dependence of Σ_a^N on E_n. The microscopic cross-section for neutron absorption by nitrogen (σ_a^N) has the form

$$\sigma_a^N(E_n) = \sigma_a^N(E_0)/(E_n/E_0)^{1/2} \tag{5}$$

where $E_0 = 0.0253$ eV and $\sigma_a^N(E_0) = 1.88$ b. Since Σ_a^N decreases with increasing E_n as $1/\sqrt{E_n}$, the magnitude of $\Phi^N(E_p)/\Phi(E_n)$ does as well. Finally, in order to compensate for the abscissa, E_p, being plotted on a logarithmic scale, in figure 1 the ordinate, $\Phi^N(E_p)$ is weighted by E_p, in order to preserve the geometrical significance of the area under the curve. With E_p plotted on a logarithmic scale and $E_p\Phi^N(E_p)$ plotted on a linear scale, an increment of area under the curve with width dE_p about E_p is proportional to the proton fluence with proton energies in dE_p about E_p.

3.2. $\mathrm{RBE}^N(E_n)$

Figure 2 is a graph of $L\Phi^N(L)$ versus the proton linear energy transfer, L, for protons in tissue. It was obtained by changing variables from E to L in the manner described by Blue *et al* (1993). As in figure 1, upon which figure 2 is based, in figure 2 it is assumed that $\Phi(E_n) = 1\ \mathrm{cm}^{-2}$ and $\rho = 1.07\ \mathrm{g\ cm}^{-3}$ with the tissue being 3.5% nitrogen by weight.

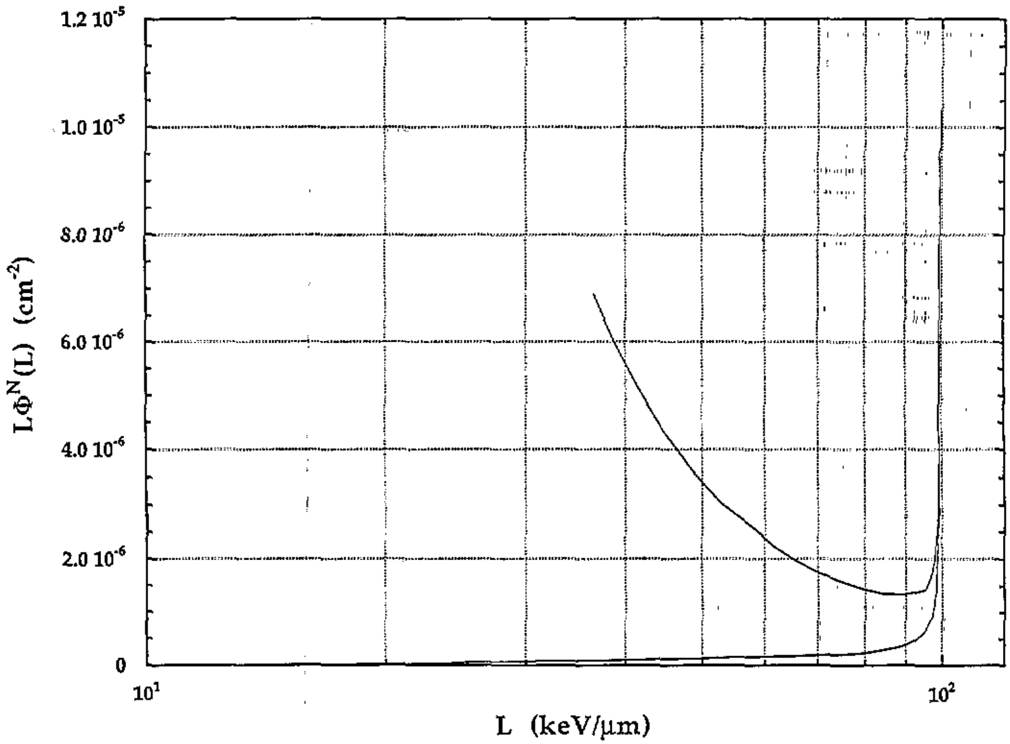

Figure 2. $L\Phi^N(L)$ (in cm^{-2}) versus L (in keV $\mu\mathrm{m}^{-1}$) for protons in tissue for thermal neutrons ($E_n = 0.0253$ eV) assuming $\Phi(E_n) = 1\ \mathrm{cm}^{-2}$.

Following Blue *et al* (1993), $\mathrm{RBE}(E_n)$ can be calculated from the following expression:

$$\mathrm{RBE}(E_n) = \int \mathrm{RBE}(L) L \frac{\Phi(L)}{\Phi(E_n)}\, dL \Big/ \int L \frac{\Phi(L)}{\Phi(E_n)}\, dL. \qquad (6)$$

This expression was numerically integrated to find $\mathrm{RBE}^N(E_n)$ by using $\Phi^N(L)/\Phi(E_n)$ as given in figure 2 and $\mathrm{RBE}(L)$ as used by Blue *et al* (1993). The resulting $\mathrm{RBE}^N(E_n)$ is equal to a constant value of 3.0, independent of the shape of $\Phi^N(L)/\Phi(E_n)$, as a consequence of the approximation we have made of constant E_p^N for all E_n.

3.3. $\mathrm{RBE}_t(E_n)$

The $\mathrm{RBE}^N(E_n)$ we have calculated thus far in this paper is appropriate for multiplication by the absorbed dose due to the $^{14}N(n,p)^{14}C$ reaction. That is, the $\mathrm{RBE}^N(E_n)$ that was calculated is an absorbed-dose-averaged RBE for energetic protons created as a consequence of (n, p) reactions with nitrogen in tissue.

In fact, the neutron absorbed dose has components due to both the $^{14}N(n,p)^{14}C$ and $^1H(n,n')^1H$ reactions. A total RBE for neutrons of energy E_n, i.e. an RBE including both the proton recoil and nitrogen components of the neutron dose, henceforth denoted $\mathrm{RBE}_t(E_n)$, is given by Blue *et al* (1993) as

$$\mathrm{RBE}_t(E_n) \equiv \{\mathrm{RBE}^N(E_n)\Sigma_a^N(E_n)E_p^N + [\mathrm{RBE}^H(E_n)\Sigma_s^H(E_n)E_n/2]\}/\{\Sigma_a^N(E_n)E_p^N + [\Sigma_s^H(E_n)E_n/2]\}. \quad (7)$$

$\mathrm{RBE}_t(E_n)$ was calculated using (7), assuming $\mathrm{RBE}^H(E_n)$ is given by Blue *et al* (1993) and $\mathrm{RBE}^N(E_n) = 3.0$, as calculated above. The macroscopic cross-section for neutron scattering on hydrogen, $\Sigma_s^H(E_n)$, was calculated using the ICRU four-element tissue approximation (ICRU 1976) and the microscopic cross-section for neutron scattering on hydrogen given by Hughes and Schwartz (1958).

3.4. Normalization of $\mathrm{RBE}_t(E_n)$ to $\langle \mathrm{RBE}_t \rangle$ measured for BMRR

Using the normalization procedure described by Blue *et al* (1993), we normalized $\mathrm{RBE}_t(E_n)$, and hence $\mathrm{RBE}^N(E_n)$ and $\mathrm{RBE}^H(E_n)$, to an average neutron RBE of 3.3, as measured for the Brookhaven Medical Research Reactor's (BMRR's) epithermal neutron beam (Gavin 1994). The normalized $\mathrm{RBE}_t(E_n)$ (which we will write as $[\mathrm{RBE}_t(E_n)]_B$, with the subscript B for Blue, to distinguish it from another expression for $\mathrm{RBE}_t(E_n)$ based on a model by Fairchild), the consistently normalized $\mathrm{RBE}^N(E_n)$, $[\mathrm{RBE}^N(E_n)]_B$, and the consistently normalized $\mathrm{RBE}^H(E_n)$, $[\mathrm{RBE}^H(E_n)]_B$, versus E_n are presented in figure 3. The normalized $\mathrm{RBE}^N(E_n)$ and $\mathrm{RBE}^H(E_n)$ are included because in BNCT, many researchers calculate the dose from the $^{14}N(n,p)^{14}C$ reaction and the $^1H(n,n')^1H$ reaction separately, and apply different (and often inconsistent) RBEs for each reaction. The neutron energy dependence of $[\mathrm{RBE}_t(E_n)]_B$, which is illustrated in figure 3, accounts for the absorbed dose from the $^1H(n,n')^1H$ and $^{14}N(n,p)^{14}C$ reactions. It does not account for other reaction channels, such as the (n,α), which open for large neutron energies; nor does it account for radiative capture in hydrogen, since it is the convention in BNCT to separately account for the transport and energy deposition of photons from this reaction.

4. Discussion

Blue *et al* (1993) presented a calculation of $\mathrm{RBE}_t(E_n)$ identical to the calculation presented here, with the exception that (as described in the introduction) $\mathrm{RBE}^N(E_n)$ in (7) for $\mathrm{RBE}_t(E_n)$ was assumed to be equal to $\mathrm{RBE}^H(E_n = 2E_p^N)$ by Blue *et al* (1993). In retrospect, this was a very good assumption. From figure 3, one can see that our rigorous approach leads to the conclusion that $[\mathrm{RBE}^N(E_n)]_B = \mathrm{RBE}^H(E_n = 960\ \mathrm{keV})]_B$ (i.e. $[\mathrm{RBE}^N(E_n)]_B$ is equal to $[\mathrm{RBE}^H(E_n)]_B$ evaluated at an E_n slightly less than $2E_p^N$).

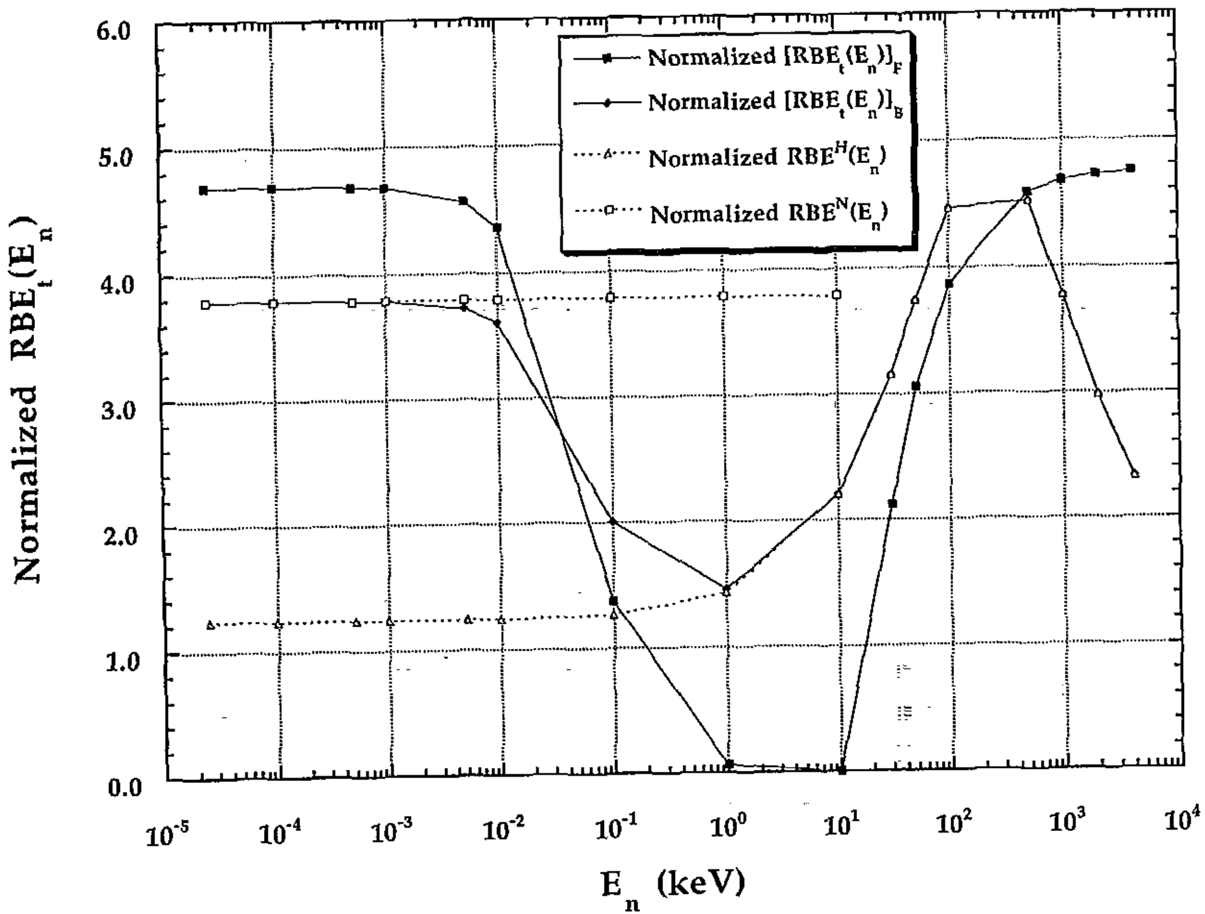

Figure 3. $[RBE_t(E_n)]_B$ and $[RBE_t(E_n)]_F$, versus E_n, normalized to measurements in the BMRR's epithermal neutron beam. Also shown are $[RBE^H(E_n)]_B$ and $[RBE^N(E_n)]_B$, the components of the normalized $[RBE_t(E_n)]_B$, versus E_n.

One can also see in figure 3 a comparison of $[\mathrm{RBE}_t(E_n)]_B$ with a calculation of $\mathrm{RBE}_t(E_n)$ derived in the appendix using a model by Fairchild and Bond (1985) for the RBE of protons. The $\mathrm{RBE}_t(E_n)$ derived using the model of Fairchild and Bond will henceforth be denoted as $[\mathrm{RBE}_t(E_n)]_F$. We chose to compare $[\mathrm{RBE}_t(E_n)]_B$ with $[\mathrm{RBE}_t(E_n)]_F$, because Fairchild was the leader of the recent resurgence of interest in BNCT in the USA, and, as such, his assumptions regarding the biological effects of protons are widely known and have been applied in many calculations. Specifically, Fairchild and Bond (1985) suggested assuming that the RBE of recoil protons is a constant with value RBE_p, from the protons' birth energies to a cut-off energy (E_c), below which the protons do not have any biological effect. Fairchild and Bond also suggested assuming for the constant RBE a value of 2.0 and for E_c a value of 10 keV. In our calculation of $[\mathrm{RBE}_t(E_n)]_F$, RBE_p and E_c are free parameters.

The expression for $[\mathrm{RBE}_t(E_n)]_F$ derived in the appendix is repeated here:

$$[\mathrm{RBE}_t(E_n)]_F = \mathrm{RBE}_p\{\Sigma_a^N(E_n)(E_p^N - E_c) + \Sigma_s^H(E_n)(E_n - E_c)^2/2E_n\}/[\Sigma_a^N(E_n)E_p^N + \Sigma_s^H(E_n)E_n/2] \qquad E_n > E_c \tag{8}$$

$$[\mathrm{RBE}_t(E_n)]_F = \mathrm{RBE}_p\{\Sigma_a^N(E_n)(E_p^N - E_c)\}/[\Sigma_a^N(E_n)E_p^N + \Sigma_s^H(E_n)E_n/2] \qquad E_n < E_c.$$

In evaluating the expression for $[\mathrm{RBE}_t(E_n)]_F$ given in (8), we assumed, as suggested by Fairchild and Bond, that $E_c = 10$ keV. However, to incorporate the results of recent radiobiology experiments into Fairchild's model, we determined RBE_p by a normalization procedure identical to the procedure that we used to normalize $[\mathrm{RBE}_t(E_n)]_B$. That is, we adjusted RBE_p so that the average neutron RBE, $\langle\mathrm{RBE}_t\rangle$, is equal to 3.3. For this average RBE, $\mathrm{RBE}_p = 4.8$.

In comparing $[\mathrm{RBE}_t(E_n)]_B$ with $[\mathrm{RBE}_t(E_n)]_F$, one sees that $[\mathrm{RBE}_t(E_n)]_F$ exhibits the same general energy dependence as $[\mathrm{RBE}_t(E_n)]_B$. The greatest discrepancy between $[\mathrm{RBE}_t(E_n)]_B$ and $[\mathrm{RBE}_t(E_n)]_F$ occurs for neutron energies larger than approximately 350 keV, since $[\mathrm{RBE}_t(E_n)]_B$ decreases for E_n greater than 350 keV, while $[\mathrm{RBE}_t(E_n)]_F$ remains constant. The overall similarity between the curves is the result of three factors. The first is that both of the expressions for $\mathrm{RBE}_t(E_n)$ were formed by combining expressions for $\mathrm{RBE}^N(E_n)$ and $\mathrm{RBE}^H(H_n)$ in the manner prescribed in (7), as can be seen by comparing (A15) with (7). Of course, the $^{14}\mathrm{N}(\mathrm{n,p})^{14}\mathrm{C}$ and the $^1\mathrm{H}(\mathrm{n,n'})^1\mathrm{H}$ reaction cross-sections used in calculating $[\mathrm{RBE}_t(E_n)]_B$ and $[\mathrm{RBE}_t(E_n)]_F$ were the same, so the denominators of the expressions for $[\mathrm{RBE}_t(E_n)]_B$ and $[\mathrm{RBE}_t(E_n)]_F$ are identical. Also, according to (5), (7) and (A15), the contributions to the numerators of $[\mathrm{RBE}_t(E_n)]_B$ and $[\mathrm{RBE}_t(E_n)]_F$ from $^{14}\mathrm{N}(\mathrm{n,p})^{14}\mathrm{C}$ reactions vary in proportion to tissue's nitrogen concentration and approximately as $1/\sqrt{E_n}$. Finally, although the contributions to the numerators of $[\mathrm{RBE}_t(E_n)]_B$ and $[\mathrm{RBE}_t(E_n)]_F$ from recoil protons are different for $[\mathrm{RBE}_t(E_n)]_B$ and $[\mathrm{RBE}_t(E_n)]_F$, due to the fact that the dependence of $\mathrm{RBE}^H(E_n)$ on E_n is different for the different models, the numerators of $[\mathrm{RBE}_t(E_n)]_B$ and $[\mathrm{RBE}_t(E_n)]_F$ are otherwise similar for the following reasons. For both models, the contribution of recoil protons to the numerator, in addition to depending on E_n through $\mathrm{RBE}^H(E_n)$, varies in proportion to the tissue's hydrogen concentration and very approximately as E_n, through its dependence on the kerma factor for the proton recoil component of the dose. Consequently, the $^{14}\mathrm{N}(\mathrm{n,p})^{14}\mathrm{C}$ reaction dominates $\mathrm{RBE}_t(E_n)$ for small E_n, for both $[\mathrm{RBE}_t(E_n)]_B$ and $[\mathrm{RBE}_t(E_n)]_F$, and the $^1\mathrm{H}(\mathrm{n,n'})^1\mathrm{H}$ reaction dominates $\mathrm{RBE}_t(E_n)$ for large E_n. The exact value of E_n for the transition depends on the nitrogen to hydrogen concentration ratio and the dependence of $\mathrm{RBE}^H(E_n)$ on E_n.

A second factor causing the overall similarity in the energy dependence of $[\mathrm{RBE}_t(E_n)]_B$ and $[\mathrm{RBE}_t(E_n)]_F$ is the similarity, between the two models, of the dependence of $\mathrm{RBE}^N(E_n)$

and $RBE^H(E_n)$ on E_n. For both models, $RBE^N(E_n)$ is a constant. Also, $RBE^H(E_n)$ depends similarly on E_n for the two models, because for both models it is assumed that the protons that are born in the $^1H(n, n')^1H$ reaction are born uniformly in energy for proton energies below E_n. A second reason for the similarity in $RBE^H(E_n)$ between the models is that Fairchild's choice of $E_c = 10$ keV was very good. Remember, in Fairchild's model, E_c is a cut-off for E_n, below which $RBE^H(E_n)$ is zero. Had E_c been chosen to be larger or smaller, the minimum of $[RBE_t(E_n)]_F$ would have occurred at greater or lesser energies, respectively.

The third and final factor causing the overall similarity in the energy dependence of $[RBE_t(E_n)]_B$ and $[RBE_t(E_n)]_F$ is that they are normalized in a similar fashion to an average neutron RBE of 3.3. In summary, the similarities between $[RBE_t(E_n)]_B$ and $[RBE_t(E_n)]_F$ are due to the physics regarding neutron interactions with hydrogen and nitrogen, which is common to their derivations, and to the good assumption by Fairchild and Bond that $E_c = 10$ keV. The reader is cautioned, however, that although, for $E_c = 10$ keV, $[RBE_t(E_n)]_B$ and $[RBE_t(E_n)]_F$ are similar in shape, the similarity does not imply that a cut-off energy in fact exists, or that its value is 10 keV.

5. Conclusion

In this paper an absorbed-dose-averaged RBE of neutrons as a function of neutron energy is presented. In developing this RBE, we utilized an empirical relation between RBE and LET and considered energy deposition from protons resulting from the $^1H(n, n')^1H$ and $^{14}N(n, p)^{14}C$ reactions. We have presented previously (Blue *et al* 1993) a similar but less rigorous, calculation of $RBE_t(E_n)$ in which we assumed that $RBE^N(E_n) = RBE^H(E_n = 2E_p^N)$. From our more rigorous calculation of $RBE_t(E_n)$, we conclude that $RBE^N(E_n) = RBE^H(E_n = 1.63E_p^N)$. In this paper, the calculated $RBE_t(E_n)$ was normalized to an average fast-neutron RBE measured in the BMRR epithermal neutron beam. The normalized RBE was compared to a calculation of the RBE as a function of neutron energy based on Fairchild's model of the RBE of recoil protons. The $RBE_t(E_n)$ based on Fairchild's model generally agrees with our calculated $RBE_t(E_n)$ in spite of the fact that the underlying assumptions of Fairchild's model regarding the biological effects of protons are significantly different from the assumptions we have made in deriving our RBE and do not seem to be physical.

Acknowledgments

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Appendix

In this appendix, we consider the implications of Fairchild's assumptions regarding the biological effects of recoil protons on the mathematical form of RBEs of neutrons with energy E_n for the proton recoil component of the dose, $RBE^H(E_n)$, the nitrogen component of the dose, $RBE^N(E_n)$, and the total dose including both the proton recoil and nitrogen components, $RBE_t(E_n)$. Specifically, Fairchild and Bond suggested (1985) assuming that the RBE of recoil protons is constant (RBE_p) from their birth energies to a cut-off energy

(E_c), below which the protons do not have any biological effect. Fairchild and Bond also suggested assuming for the constant RBE_p a value of 2.0 and for E_c a value of 10 keV. More recently, Alpen has reviewed the proton data and suggested that Fairchild's model for the biological effects of recoil protons be adopted, with an assumed constant RBE_p of approximately 3.5 and $E_c = 10$ keV (Alpen 1992). To determine the implications of Fairchild's model on the mathematical forms of $\mathrm{RBE}^{\mathrm{H}}(E_n)$, $\mathrm{RBE}^{\mathrm{N}}(E_n)$ and $\mathrm{RBE}_t(E_n)$, we first express $\mathrm{RBE}^{\mathrm{H}}(E_n)$ as the ratio of the RBE dose (H^{H}) to the absorbed dose (D^{H}) for a monoenergetic fluence of neutrons with neutron energy E_n; i.e.

$$\mathrm{RBE}^{\mathrm{H}}(E_n) = H^{\mathrm{H}}/D^{\mathrm{H}} \tag{A1}$$

where

$$H^{\mathrm{H}} = 1.6 \times 10^{-8}[\Phi(E_n)/\rho]\Sigma_s^{\mathrm{H}}(E_n) \int_{E_c}^{E_n} p(E_n \to E_p)(E_p - E_c)\mathrm{RBE}(E_p)\,\mathrm{d}E_p \tag{A2}$$

and

$$D^{\mathrm{H}} = 1.6 \times 10^{-8}[\Phi(E_n)/\rho]\Sigma_s^{\mathrm{H}}(E_n) \int_{0}^{E_n} p(E_n \to E_p)E_p\,\mathrm{d}E_p. \tag{A3}$$

In (A2) and (A3), H^{H} is in units of RBE cGy, D^{H} is in units of cGy, $\Sigma_s^{\mathrm{H}}(E_n)$ is the macroscopic cross-section in units of cm^{-1} for neutrons scattering with hydrogen in tissue, $\Phi(E_n)$ is in units of neutrons cm^{-2}, E_p and E_c are in units of MeV, ρ is the mass density of tissue in units of g cm^{-3}, $p(E_n \to E_p)\,\mathrm{d}E_p$ is the probability that a neutron of energy E_n will, in a single collision, create a recoil proton with energy in $\mathrm{d}E_p$ about E_p and $\mathrm{RBE}(E_p)$ is the relative biological effectiveness of a recoil proton with birth energy E_p averaged over its pathlength to the cut-off energy. Assuming

$$p(E_n \to E_p) = 1/E_n \tag{A4}$$

and performing the integral in (A3),

$$D^{\mathrm{H}} = 1.6 \times 10^{-8}[\Phi(E_n)/\rho]\Sigma_s^{\mathrm{H}}(E_n)E_n/2. \tag{A5}$$

Assuming further that

$$\begin{aligned} &\mathrm{RBE}(E_p) = \mathrm{RBE}_p \qquad E_n > E_c \\ &\mathrm{RBE}(E_p) = 0 \qquad E_n < E_c \end{aligned} \tag{A6}$$

and performing the integral in (A2)

$$H^{\mathrm{H}} = 1.6 \times 10^{-8}[\Phi(E_n)/\rho]\Sigma_s^{\mathrm{H}}(E_n)\mathrm{RBE}_p(E_n - E_c)^2/(2E_n). \tag{A7}$$

Finally, from (A1), (A5) and (A7)

$$\begin{aligned} &\mathrm{RBE}^{\mathrm{H}}(E_n) = \mathrm{RBE}_p(E_n - E_c)^2/E_n^2 \qquad E_n > E_c \\ &\mathrm{RBE}^{\mathrm{H}}(E_n) = 0 \qquad E_n < E_c. \end{aligned} \tag{A8}$$

Equation (A8) for $\text{RBE}^{\text{H}}(E_n)$ is based upon the ratio of H to D for energy depositions by recoil protons. However, neutrons can interact to produce energetic protons in a second reaction, the $^{14}\text{N}(\text{n, p})^{14}\text{C}$ reaction. When the thermal neutrons induce this reaction, the protons are emitted with a kinetic energy E_p^{N} (in units of MeV). The calculation of $\text{RBE}^{\text{N}}(E_n)$ for the $^{14}\text{N}(\text{n, p})^{14}\text{C}$ reaction proceeds in a manner similar to that leading to (A8). As before, we first express $\text{RBE}^{\text{N}}(E_n)$ as the ratio of the RBE dose (H^{N}) to the absorbed dose (D^{N}) for neutrons of energy E_n interacting with nitrogen. That is, we assume

$$\text{RBE}^{\text{N}}(E_n) = H^{\text{N}}/D^{\text{N}}. \tag{A9}$$

In the case of neutron interactions with nitrogen, the expressions for H^{N} and D^{N} are slightly modified from (A2) and (A3) (for H^{H} and D^{H}, respectively) by replacing Σ_s^{H} with Σ_a^{N}, yielding

$$H^{\text{N}} = 1.6 \times 10^{-8}[\Phi(E_n)/\rho]\Sigma_a^{\text{N}}(E_n) \int_0^{E_n} p(E_n \to E_p)(E_p - E_c)\text{RBE}(E_p)\,\text{d}E_p \tag{A10}$$

and

$$D^{\text{N}} = 1.6 \times 10^{-8}[\Phi(E_n)/\rho]\Sigma_a^{\text{N}}(E_n) \int_0^{\infty} p(E_n \to E_p)E_p\,\text{d}E_p. \tag{A11}$$

Also, in (A10) and (A11), the limits of integration are changed, since the energy dependence of the emitted proton on E_n is entirely specified in $p(E_n \to E_p)$, whereas for (A2) and (A3), the relationship between E_n and E_p is expressed by $p(E_n \to E_p)$ and the limits of integration on E_p. Finally, the interpretation of $p(E_n \to E_p)\,\text{d}E_p$ in (A10) and (A11) is modified from its interpretation in (A2) and (A3) to be the probability that a neutron with energy E_n will, in an absorption reaction with nitrogen, create a proton with energy in $\text{d}E_p$ about E_p. Since the energies of the neutrons likely to induce a $^{14}\text{N}(\text{n, p})^{14}\text{C}$ reaction are very much smaller than the kinetic energies of the emitted protons, we assume that E_n can be neglected relative to E_p. Then, $p(E_n \to E_p) = \delta(E_p - E_p^{\text{N}})$.

Performing the integral in (A11),

$$D^{\text{N}} = 1.6 \times 10^{-8}[\Phi(E_n)/\rho]\Sigma_a^{\text{N}}(E_n)E_p^{\text{N}}. \tag{A12}$$

Assuming further that $\text{RBE}(E_p)$ is a constant for proton energies greater than E_c and zero otherwise, as specified in (A6), and performing the integral in (A10),

$$H^{\text{N}} = 1.6 \times 10^{-8}[\Phi(E_n)/\rho]\Sigma_a^{\text{N}}(E_n)(E_p^{\text{N}} - E_c)\text{RBE}_p. \tag{A13}$$

Finally, from (A1), (A12) and (A13)

$$\text{RBE}^{\text{N}}(E_n) = [(E_p^{\text{N}} - E_c)/E_p^{\text{N}}]\text{RBE}_p \tag{A14}$$

a constant, for energy depositions due to $^{14}\text{N}(\text{n, p})^{14}\text{C}$ reactions.

In summary, (A8) describes the dependence of $\text{RBE}^{\text{H}}(E_n)$ on E_n for proton recoil interactions, and (A14) describes the dependence of $\text{RBE}^{\text{N}}(E_n)$ on E_n for $^{14}\text{N}(\text{n, p})^{14}\text{C}$ reactions. To combine (A8) and (A14) to form a single expression for energy depositions due to neutron interactions, we use (A15):

$$\text{RBE}_t(E_n) \equiv [\text{RBE}^{\text{N}}(E_n)k_n^{\text{N}}(E_n) + \text{RBE}^{\text{H}}(E_n)k_n^{\text{H}}(E_n)]/[k_n^{\text{N}}(E_n) + k_n^{\text{H}}(E_n)]. \tag{A15}$$

In (A15), k_n^N and k_n^H are the neutron kerma factors for the nitrogen and proton recoil components of the absorbed dose, respectively, i.e.

$$k_n^N(E_n) \cong b\Sigma_a^N(E_n)E_p^N \tag{A16}$$

and

$$k_n^H(E_n) \cong b\Sigma_s^H(E_n)E_n/2 \tag{A17}$$

where b is a unit conversion factor. Substituting (A8), (A14), (A16) and (A17) into (A15) yields

$$\begin{aligned}\mathrm{RBE}_t(E_n) &= \mathrm{RBE}_p\{\Sigma_a^N(E_n)(E_p^N - E_c) + \Sigma_s^H(E_n)(E_n - E_c)^2/2E_n\}/[\Sigma_a^N(E_n)E_p^N \\ &\quad + \Sigma_s^H(E_n)E_n/2] \qquad E_n > E_c \qquad\qquad \text{(A18)} \\ \mathrm{RBE}_t(E_n) &= \mathrm{RBE}_p\{\Sigma_a^N(E_n)(E_p^N - E_c)\}/[\Sigma_a^N(E_n)E_p^N + \Sigma_s^H(E_n)E_n/2] \qquad E_n < E_c.\end{aligned}$$

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